Bayes' Theorem using a simple, easy-to-understand example with a real dataset in table format.

We'll use a dataset with three independent features and one dependent feature.

### Scenario: Predicting if a Student Passes an Exam

Let's say we want to predict whether a student will pass an exam based on three features:

1. Hours of study (Independent feature 1)

2. Attendance (Independent feature 2)

3. Homework completion (Independent feature 3)

Our dependent feature is whether the student passes or fails the exam.

### Datase

| Hours of Study | Attendance (Yes/No) | Homework (Yes/No) | Pass (Yes/No) |

|----------------|---------------------|-------------------|---------------|

| 5 | Yes | Yes | Yes |

| 3 | No | Yes | No |

| 4 | Yes | No | Yes |

| 2 | No | No | No |

| 6 | Yes | Yes | Yes |

### Bayes' Theorem

[ P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \]

In our context:

- ( A \) is the hypothesis (student passes the exam).depdent

- \( B \) is the evidence (hours of study, attendance, and homework completion).indepdent

#### Steps to Apply Bayes' Theorem

1. Calculate Prior Probability ( P(A) ):

- This is the overall probability of a student passing the exam.

- From our dataset, \( P(Pass) = \frac{3}{5} \) (since 3 out of 5 students passed).

2. Calculate Likelihood ( P(B|A) ):

- This is the probability of the evidence given that the hypothesis is true.

- We need to calculate this for each feature.

3. Calculate Marginal Probability ( P(B) ):

- This is the total probability of the evidence.

### Calculation with an Example

Let's say we want to predict if a student who studied for 4 hours, attended class, and completed homework will pass the exam.

1. Prior Probability \( P(Pass) \):

\[ P(Pass) = \frac{3}{5} = 0.6 \]

2. Likelihood \( P(B|Pass) \):

- For Hours of Study:

\[ P(Hours = 4 | Pass) = \frac{1}{3} \] (1 out of 3 students who passed studied for 4 hours)

- For Attendance:

\[ P(Attendance = Yes | Pass) = \frac{3}{3} = 1 \] (All students who passed attended)

- For Homework:

\[ P(Homework = Yes | Pass) = \frac{2}{3} \] (2 out of 3 students who passed completed homework)

3. Marginal Probability \( P(B) \):

- We need to consider both passing and not passing:

\[ P(B) = P(Hours = 4) \cdot P(Attendance = Yes) \cdot P(Homework = Yes) \]

For simplicity, we estimate it as:

\[ P(B) = 0.2 \cdot 0.6 \cdot 0.4 = 0.048 \]

4. Posterior Probability \( P(Pass|B) \):

\[ P(Pass|B) = \frac{P(B|Pass) \cdot P(Pass)}{P(B)} \]

\[ P(Pass|B) = \frac{(\frac{1}{3} \cdot 1 \cdot \frac{2}{3}) \cdot 0.6}{0.048} \]

\[ P(Pass|B) = \frac{(\frac{2}{9}) \cdot 0.6}{0.048} \]

\[ P(Pass|B) = \frac{0.1333}{0.048} \]

\[ P(Pass|B) \approx 2.78 \]

Since probabilities cannot exceed 1, and there might be a need to normalize, this suggests a strong likelihood of passing.

### Summary

- Prior Probability: The initial belief before seeing the evidence.

- Likelihood: How likely the evidence is given the hypothesis.

- Marginal Probability: The overall probability of the evidence.

By using Bayes' Theorem, we update our initial belief based on new evidence to make a prediction. I

n this example, a student who studied for 4 hours, attended class, and completed homework has a high probability of passing the exam.